Exam PPP

6 April 2016

- Put your name and student number on each answer sheet.
- Answer all questions short and to the point, but complete; write legible.
- Final point grade = total number of 9*(total points/100) + 1

1. **Bottomonium (20 points)**
   Bottomonium is a meson that is composed of a heavy b quark and a heavy $\bar{b}$ anti-quark. The bare mass of the b ($\bar{b}$) quark is 4.6 GeV. Figure 1 illustrates the mass spectrum of bottomonium. Bottomonium (similar to charmonium) belongs to the family of *quarkonium* and it can be produced experimentally via the annihilation of electrons with positrons. You should only consider the strong and e.m. interactions and ignore weak interactions in this system.

   a) Sketch the dominant Feynman diagram of the production of a bottomonium state in electron-positron annihilation. Explain why pre-dominantly $J^{PC} = 1^{--}$ states, a.k.a. as Υ, are produced directly in this annihilation process.

   b) Which orbital angular momenta (L) are allowed between the quark and antiquark of the produced $J^{PC} = 1^{--}$ states? Motivate your answer.

   c) The dominant decay mode of the Υ is the process $\Upsilon \to ggg$ with g as the gluon. The gluons eventually will hadronize. The decay into two gluons is not possible because of the conservation of charge conjugation. Explain why the Υ cannot decay into one gluon either.

   d) The widths of the three lowest lying Υ states (with masses between 9.5 and 10.3 GeV and in Fig. 1 indicated by $\Upsilon(1S, 2S, 3S)$) are about 20-54 keV, whereas the width of the next radially excited $\Upsilon(4S)$ state, with a mass of about 10.6 GeV, is about 20 MeV, e.g. a factor 10^3 larger. Estimate the corresponding lifetimes and explain why the width suddenly increases. [Hint: the lightest open-bottom meson (B) is 5.3 GeV.]

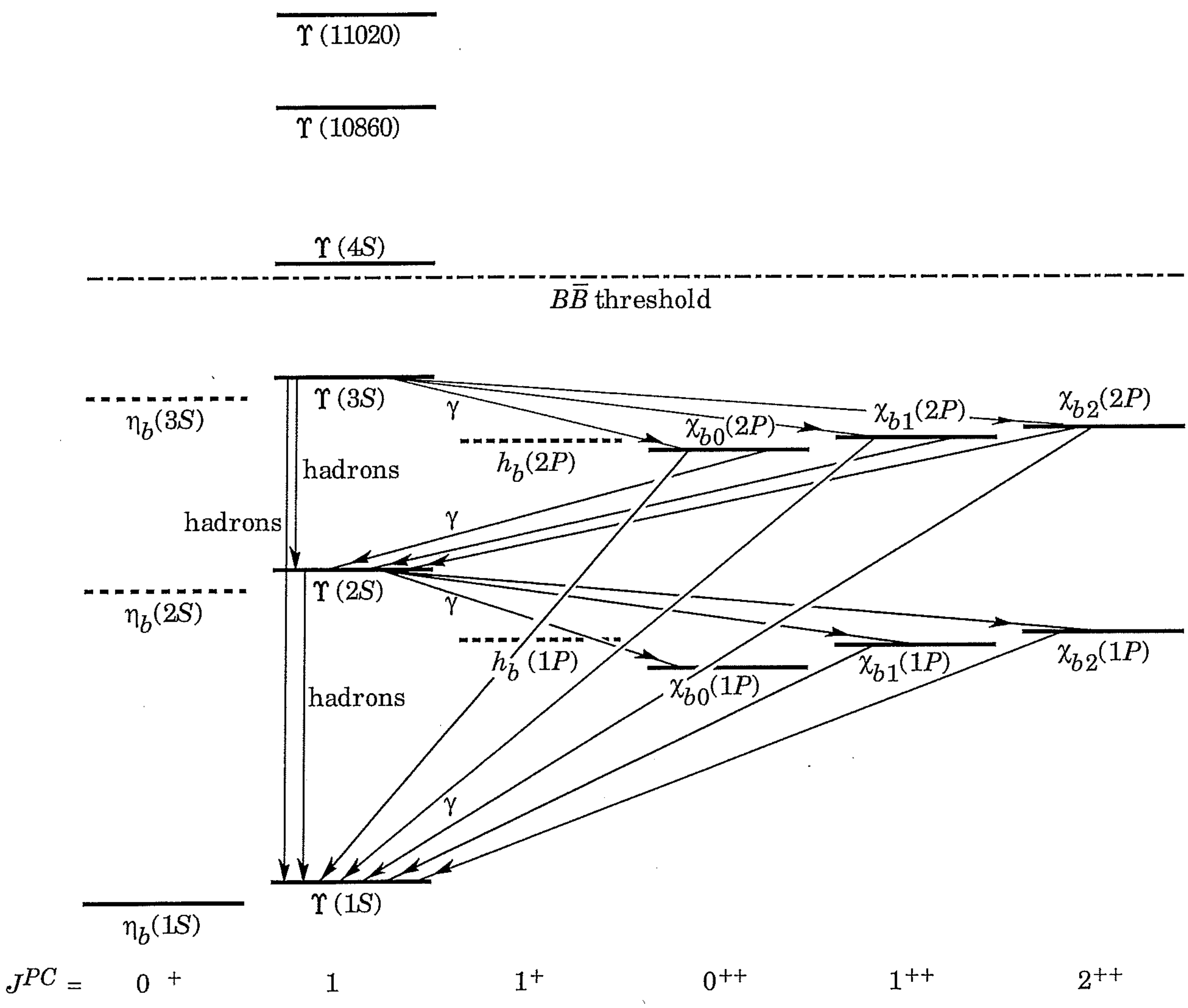

Figure 1: The mass spectrum of bottomonium states. The Υ states have a spin-parity of $J^{PC} = 1^{--}$.

2. **The Higgs phenomena (20 points)**

Consider a Lagrangian density for a complex scalar field, ϕ, by $L = \frac{1}{2}\partial_\mu \phi^* \partial^\mu \phi - V(\phi^*\phi)$. Here, the derivative $\partial_\mu = (\frac{\partial}{\partial t}, \frac{\partial}{\partial \vec{x}})$, the potential $V = \frac{\mu^2}{2}\phi^*\phi + \frac{\lambda}{4}(\phi^*\phi)^2$ with constants μ^2 and λ. The complex scalar field can be written in polar coordinates $\phi = \rho e^{i\alpha}$ with real scalar fields $\rho(x)$, $\alpha(x)$ that depend on time-space coordinate $x_\mu = (t, \vec{x})$. Note that L obeys global U(1) symmetry, e.g. L is invariant under the transformation $\phi \to e^{i\theta}\phi$ with θ a constant, independent on x.

a) Consider the case in which $\mu^2 < 0$ and $\lambda > 0$. Explain why this condition leads to *spontaneous symmetry breaking* (SSB).

b) Demonstrate or argue that the SSB case leads to a massless Goldstone boson associated with the α field and a massive boson (Higgs) associated with the ρ field.

The Lagrangian L is <u>not</u> Gauge (local U(1)) invariant, since L is not invariant under the transformation $\phi \to e^{i\theta(x)}\phi$ where $\theta(x)$ varies from place-to-place. One can, for good reasons, modify the Lagrangian such that it obeys Gauge invariance by 1) adding an additional vector field A_μ with the transformation property ($A_\mu \to A_\mu - \partial_\mu \theta$), 2) replacing the derivative ∂_μ by a so-called covariant derivative ($D_\mu = \partial_\mu + iA_\mu$) which leads naturally to the interaction of the ϕ field with the vector field A_μ, and 3) add the dynamics of A_μ. This eventually gives a Lagrangian of the form $L_{\text{Gauge}} = D_\mu \phi D^\mu \phi^* - V(\phi\phi^*) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

c) Explain why one cannot <u>explicitly</u> add a mass term, $\frac{m^2}{2}A^2$, for the new field A_μ in the Lagrangian? Show or argue that in the case of SSB, the new field A_μ will obtain a mass afterall.

d) In the real world, the quanta of the field A correspond to the weak force carriers, the W bosons, whereby ϕ represents the Higgs field. Imagine a world in which the Higgs field has the property $\mu^2 > 0$ and $\lambda \geq 0$. In this unbroken case, the quanta of A_μ will be massless. What will happen to the mass of the <u>fermions</u> (quarks and leptons) in the Standard Model? Motivate your answer.

3. **Pion-nucleon scattering (20 points)**

Consider a beam of charged pions that scatter elastically on a proton target at rest, e.g. $\pi^{+,-} + p \to \pi^{+,-,0} + N$ where N indicates a nucleon (proton or neutron). The energy of the pion beam is chosen such that the reaction takes place predominantly via the formation of a $\Delta(1232)$ resonance, e.g. $\pi + N \to \Delta(1232) \to \pi + N$. The isospins of the Δ, pion, and nucleon are 3/2, 1, and 1/2, respectively. The spin-parity of the Δ, pion, and nucleon are J^P=3/2$^+$, 0^-, and 1/2$^+$, respectively.

a) Explain qualitatively how the experimental discovery of the Δ played a crucial role in the existence of *color*, and, thereby, the basis of QCD.

b) Calculate the incident pion-beam momentum that corresponds to the production of the Δ resonance at its peak position ($M(\Delta) = 1232$ MeV). The pion (proton) mass is 140 MeV (940 MeV).

c) Show that the cross section ratio

$$\frac{\sigma(\pi^+ + p \to \pi^+ + p)}{\sigma(\pi^- + p \to \pi^0 + n)} = \frac{9}{2},$$

in the case of isospin conservation and under the assumption that the reaction takes place solely via the intermediate Δ resonance. Consult the table with Clebsch-Gordan coefficients; see Fig. 2.

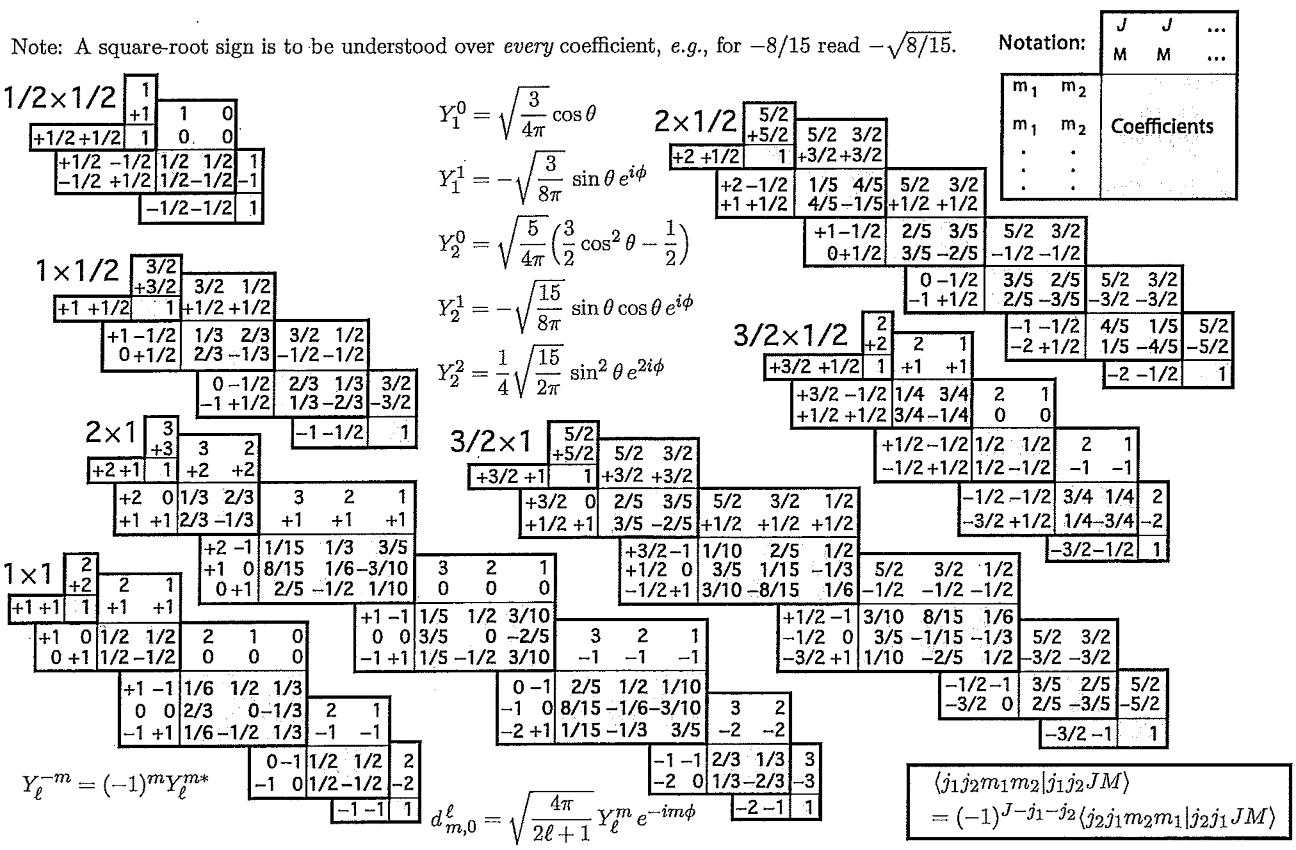

Figure 2: Clebsch-Gordan coefficients.

4. **D- and B-mesons (25 points)**

The D-mesons contain a charm-quark, together with one of the lighter quarks ($\mathrm{D}^0 : c\bar{u}, \mathrm{D}^+ : c\bar{d}, \mathrm{D}_s^+ :$ $c\bar{s}$). B-mesons contain a bottom quark instead of a charm quark ($\mathrm{B}^- : b\bar{u}, \bar{\mathrm{B}}^0 : b\bar{d}, \bar{\mathrm{B}}_s^0 : b\bar{s}$).

a) Sketch the Feynman diagrams for the decays $\mathrm{B}^- \to \mathrm{D}^0\mathrm{K}^-$ and $\mathrm{B}^- \to \bar{\mathrm{D}}^0\mathrm{K}^-$ ($\mathrm{K}^- : \bar{u}s$).

b) In each case, indicate which CKM elements are involved and explain why the D^0 is more likely to be produced than the $\bar{\mathrm{D}}^0$ (in fact, about 100 times more likely).

c) Is that also the case for the decay of a postive B to a D and K? Why (not)?

d) The D-meson has a mass of almost $1900\,\mathrm{MeV}/c^2$. Explain if the branching fraction for semi-leptonic decay into muons is much larger, much smaller, or about equal to that for decay into electrons.

e) Explain how you can use the lepton charge in the semi-leptonic decay of the D-meson to find out whether you had a D^0 or $\bar{\mathrm{D}}^0$ (which charge corresponds to which?).

f) Formulate the CP=+1 and CP=-1 eigenstates (respectively $|\mathrm{D}_1^0\rangle$ and $|\mathrm{D}_2^0\rangle$) using the $|\mathrm{D}^0\rangle$ and $|\bar{\mathrm{D}}^0\rangle$ states. Assume that only D^0 are produced. What is the inital state in terms of D_1^0 and D_2^0?

Magnitudes of the CKM matrix elements:

$$\begin{bmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{bmatrix} = \begin{bmatrix} 0.97427 \pm 0.00015 & 0.22534 \pm 0.00065 & 0.00351^{+0.00015}_{-0.00014} \\ 0.22520 \pm 0.00065 & 0.97344 \pm 0.00016 & 0.0412^{+0.0011}_{-0.0005} \\ 0.00867^{+0.00029}_{-0.00031} & 0.0404^{+0.0011}_{-0.0005} & 0.999146^{+0.000021}_{-0.000046} \end{bmatrix}.$$

5. **Neutrinos (15 points)**

a) Discuss two similarities and two difference between the PMNS and CKM matrix.

b) In 2015 the Nobel prize for physics was awarded to Takaaki Kajita and Arthur McDonald "for the discovery of neutrino oscillations, which shows that neutrinos have mass". Discuss why neutrino oscillations indeed prove that neutrinos have mass, and indicate what other condition(s) must be met.

c) Charged pions and kaons (mainly) decay in a muon and a muon-neutrino. Explain how this decay can be used to measure the muon neutrino mass. Would you prefer to use the pion or the kaon? Why?